

**Quiz 8**  
**Chemical Engineering Thermodynamics**  
**March 10, 2016**

**8.14.** A 1 m<sup>3</sup> isolated chamber with rigid walls is divided into two compartments of equal volume. The partition permits transfer of heat. One side contains a nonideal gas at 5 MPa and 300 K and the other side contains a perfect vacuum. The partition is ruptured, and after sufficient time for the system to reach equilibrium, the temperature and pressure are uniform throughout the system. The objective of the problem statements below is to find the final  $T$  and  $P$ .

The gas follows the equation of state

$$\frac{PV}{RT} = 1 + \left(b - \frac{a}{T}\right) \frac{P}{RT}$$

where  $b = 20 \text{ cm}^3/\text{mole}$ ;  $a = 40,000 \text{ cm}^3\text{K}/\text{mole}$ ; and  $C_p = 41.84 + 0.0847T(\text{K})$  J/mol-K.

- a. Set up and simplify the energy balance and entropy balance for this problem.
- b. Derive formulas for the departure functions required to solve the problem.
- c. Determine the final  $P$  and  $T$ .

For part “c” calculate the needed equation(s) and parameter(s).

- d. Give a series of steps necessary to solve for  $T_f$  and  $P_f$  using Solver<sup>®</sup> in Excel<sup>®</sup>.
- e. What are the values for  $T_f$  and  $P_f$  if the gas were an ideal gas? (These should be your starting values for Solver<sup>®</sup> in part “d”.)

$$R = 8.314 \text{ J/mole-K} = 8.314 \text{ cm}^3 \text{ MPa/mole-K}$$

$$\frac{(U - U^{ig})}{RT} = \int_0^{\rho} -T \left[ \frac{\partial Z}{\partial T} \right]_{\rho} \frac{d\rho}{\rho} \quad 8.22$$

$$\frac{(S - S^{ig})}{R} = \int_0^{\rho} \left[ -T \left[ \frac{\partial Z}{\partial T} \right] - (Z - 1) \right] \frac{d\rho}{\rho} + \ln Z \quad 8.23$$

$$\frac{(H - H^{ig})}{RT} = \int_0^{\rho} -T \left[ \frac{\partial Z}{\partial T} \right]_{\rho} \frac{d\rho}{\rho} + Z - 1 \quad 8.24$$

$$\frac{(A - A^{ig})}{RT} = \int_0^{\rho} \frac{(Z - 1)}{\rho} d\rho - \ln Z \quad 8.25$$

$$\frac{(G - G^{ig})}{RT} = \int_0^{\rho} \frac{(Z - 1)}{\rho} d\rho + (Z - 1) - \ln Z \quad 8.26$$

$$\left( \frac{H - H^{ig}}{RT} \right) = - \int_0^P T \left( \frac{\partial Z}{\partial T} \right)_P \frac{dP}{P} \quad 8.29$$

$$\left( \frac{S - S^{ig}}{R} \right) = - \int_0^P \left[ (Z - 1) + T \left( \frac{\partial Z}{\partial T} \right)_P \right] \frac{dP}{P} \quad 8.30$$

$$H = U + PV \Rightarrow \frac{H - H^{ig}}{RT} = \frac{U - U^{ig}}{RT} + \frac{PV - RT}{RT} = \frac{U - U^{ig}}{RT} + Z - 1$$

$$A = U - TS \Rightarrow \frac{A - A^{ig}}{RT} = \frac{U - U^{ig}}{RT} - \frac{S - S^{ig}}{R}$$

8.20

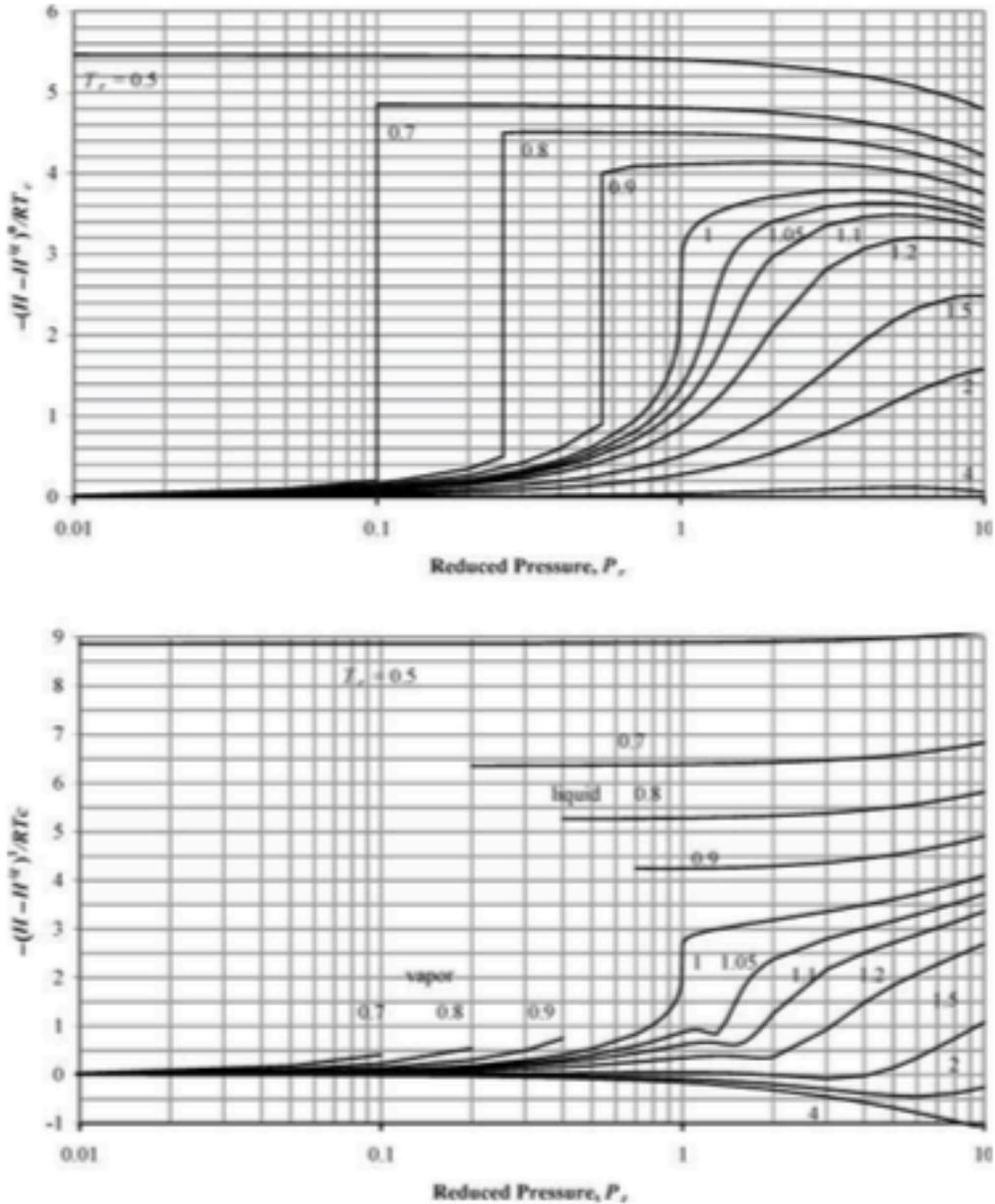
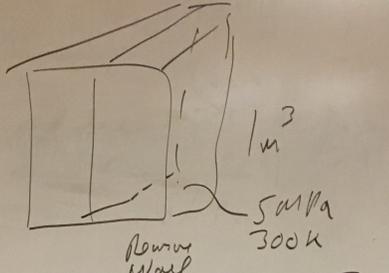


Figure 8.7. Generalized charts for estimating  $(H - H^{ig})/RT_c$  using the Lee-Kesler equation of state.  $(H - H^{ig})^0/RT_c$  uses  $\omega = 0.0$ , and  $(H - H^{ig})^1/RT_c$  is the correction factor for a hypothetical compound with  $\omega = 1.0$ . Divide by reduced temperature to obtain the enthalpy departure function.

**ANSWERS: Quiz 8**  
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0.14



$PV = nRT$   
 $V$   
 $P \sim \frac{1}{V}$

$T_f, P_f = ?$   
 $Z = 1 + \left(b - \frac{a}{T}\right) \frac{P}{RT}$   
 $b = 20 \frac{\text{cm}^3}{\text{mole}}$   
 $a = 40,000 \frac{\text{cm}^3 \text{K}}{\text{mole}}$   
 $C_p = 41.8 + 0.089 T \quad (\text{J/molK})$

d) Energy Balance  
 $\Delta U = 0$   
 Entropy Bal.  $\rightarrow 0$   
 $\Delta S = S_{gen}$

(b) 1) want  $U_{inhal}$   
 ideal

$\frac{U - U^{is}}{RT} = \frac{H - H^{is}}{RT} - (Z - 1) \quad \text{P.308}$   
 $\frac{H - H^{is}}{RT} = - \int_0^P \frac{1}{T} \left(\frac{\partial Z}{\partial T}\right)_P \frac{dP}{P} \quad \text{P.310}$   
 $\left(\frac{\partial Z}{\partial T}\right)_P = -\left(b - \frac{a}{T}\right) \frac{P}{RT^2}$   
 $\left(\frac{H - H^{is}}{RT}\right) = \frac{\left(b - \frac{a}{T}\right) P}{RT}$   
 $\left(\frac{U - U^{is}}{RT}\right) = -\frac{a}{RT^2} P$

c)

$U^i = (U - U^i) + \int_{T=300K}^{T=300K} (C_p - R) dT$   
 $= \frac{-aP}{T} = -\frac{40,000 \frac{\text{cm}^3 \text{K}}{\text{mole}} \text{J MPa}}{300K}$   
 $U^i = -677 \frac{\text{J}}{\text{mole}} = U^f$

$0 = 677 \frac{\text{J}}{\text{mole}} + \frac{-40,000 \frac{\text{cm}^3 \text{K}}{\text{mole}} \left(\frac{P_f}{T_f}\right)}{T_f} + \frac{41.8T_f + 0.089T_f^2}{20} \left(\frac{P_f}{T_f}\right) \left(\frac{1}{T_f}\right)$

Equation 1

$F(T, P)$	0	= 0
$\rightarrow$	300 K	= 0
$P$	5/2 MPa	(initial value)

$Z_i = 1 + \left(b - \frac{a}{T_i}\right) \frac{P_i}{RT_i} = 0.71$   
 $V_i = \frac{Z_i RT_i}{P_i} = 385.7 \frac{\text{cm}^3}{\text{mole}}$   
 $V_f = 2V_i = 771 \frac{\text{cm}^3}{\text{mole}} = 1 + \left(b - \frac{a}{T_f}\right) \frac{P_f}{RT_f}$

Equation 2

Initial guess for ideal gas would be  $T_f = 300\text{K}$  and  $P_f = 5\text{MPa}/2$ .

**3.14) An 1-m<sup>3</sup> isolated chamber with rigid walls is divided into two...**

a) Energy balance

$$\Delta U = 0$$

Entropy balance

$$\Delta \underline{S} = \Delta \underline{S}_{gen}$$

b) Need departure function for U. First get departure for H and use  $H = U + PV$

$$\frac{H - H^{ig}}{RT} - Z + 1 = \frac{U - U^{ig}}{RT}$$

$$Z = 1 + \frac{bP}{RT} - \frac{aP}{RT^2}$$

$$-T \left( \frac{\partial Z}{\partial T} \right)_P = \frac{bT}{RT^2} - \frac{2aPT}{RT^3}$$

$$\frac{H - H^{ig}}{RT} = \int_0^P \left( \frac{bP}{RT} - \frac{2aP}{RT^2} \right) \frac{dP}{P} = \frac{bP}{RT} - \frac{2aP}{RT^2}$$

$$1 - Z = - \left( b - \frac{a}{T} \right) \frac{P}{RT}$$

$$U - U^{ig} = \frac{RT}{RT} \left[ \left( bP - \frac{2aP}{T} \right) - bP + \frac{aP}{T} \right]$$

$$U - U^{ig} = - \frac{aP}{T}$$

c) Set  $U_R^{ig} = 0$ , set  $T_R = 300$  K and  $P_R = 2$  MPa

$$U^i = (U - U^{ig})^i + \int_{T_R}^{T^i} (C_p - R) dT = -40000(5)/300 = -667 \text{ J/mol}$$

$$U^f = (U - U^{ig})^f + \int_{T_R}^{T^f} (C_p - R) dT = U^i = -40000(P)/(T) + 33.53 (T - 300) + 0.042 (T^2 - 300^2) = -667 \text{ J/mol}$$

$$Z^i = 0.773, V^i = ZRT/P = 0.773(8.314)(300)/5 = 385.5 \text{ cm}^3/\text{mol}$$

$$V^f = \frac{RT}{P} + \left( b - \frac{a}{T} \right) = 2V^i = 771 \text{ cm}^3/\text{mol}$$

Programming in Excel and solving,

Solve two equations and two unknowns.

$$\text{obj 1} = -40000(P)/(T) + 33.53 (T - 300) + 0.042 (T^2 - 300^2) + 667$$

$$\text{obj 2} = 8.314T/P + b - a/T - 771$$

Solver gives  $T = 295$  K,  $P = 2.76$  MPa

	final	initial
P	2.766503	5
T	295.0177	300
V	771.0133	385.5067
Z	0.869631	0.772806
U		-666.6667

$$\text{obj 1} = -2.36\text{E-}09$$

$$\text{obj 2} = -2.25\text{E-}07$$